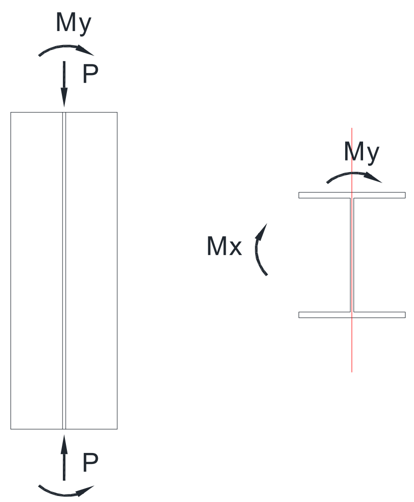
Design of W-Shapes Subjected to Axial Compression and Moments including the Second Order Effect



Materials

Grade:	SEL("Material/ASTM"; NAME;)	=	A992
F_y=	TAB("Material/ASTM";F_y;NAME=Grade)	=	50 ksi
E=			29000 ksi

Beam Length and C_b (The member is not subjected to sidesway)

Unsupported length, L_b=	14.00 ft
Strong axis unbraced length, kL_{in}=	14.00 ft
Weak axis and torsional unbraced length, kL_{out}=	14.00 ft
From Table 3-1 (AISC), C_b=	1.14

Given Straining Actions (Not including second-order effects)

P_D=	5.0 kips
P_L=	15.0 kips
M_{xD}=	15.0 kip*ft
M_{xL}=	45.0 kip*ft
M_{yD}=	2.0 kip*ft
M_{yL}=	6.0 kip*ft

From Chapter 2 of ASCE/SEI 7, the required strength (not considering second-order effects) is:

P_u=	$1.2*P_D+1.6*P_L$	=	30.0 kips
M_{ux1}=	$1.2*M_{xD}+1.6*M_{xL}$	=	90.0 kip*ft
M_{uy1}=	$1.2*M_{yD}+1.6*M_{yL}$	=	12.0 kip*ft

Section Details

sec.:	SEL("AISC/W";NAME;)	=	W10X33
depth, d=	TAB("AISC/W";d;NAME=sec.)	=	9.73 in

Web th., t_w=	TAB("AISC/W";t_w;NAME=sec.)	=	0.29 in
Flange width, b_f=	TAB("AISC/W";b_f;NAME=sec.)	=	7.96 in
Flange th., t_f=	TAB("AISC/W";t_f;NAME=sec.)	=	0.44 in
Gross Area, A=	TAB("AISC/W";A;NAME=sec.)	=	9.71 in^2
I_x=	TAB("AISC/W";I_x;NAME=sec.)	=	171.00 in^4
I_y=	TAB("AISC/W";I_y;NAME=sec.)	=	36.60 in^4

(I_x and I_y are the moment of inertia about x-and y-axes, respectively)

Plastic sec. modulus, Z_x=	TAB("AISC/W";Z_x;NAME=sec.)	=	38.80 in^3
Elastic sec. modulus, S_x=	TAB("AISC/W";S_x;NAME=sec.)	=	35.00 in^3
Plastic sec. modulus, Z_y=	TAB("AISC/W";Z_y;NAME=sec.)	=	14.00 in^3
Elastic sec. modulus, S_y=	TAB("AISC/W";S_y;NAME=sec.)	=	9.20 in^3
r_x=	TAB("AISC/W";r_x;NAME=sec.)	=	4.19 in
r_y=	TAB("AISC/W";r_y;NAME=sec.)	=	1.94 in

(r_x and r_y are the radius of gyration about x- and y- axis, respectively)

Torsional constant, J=	TAB("AISC/W";J;NAME=sec.)	=	0.58 in^4
r_{ts}=	TAB("AISC/W";r_{ts};NAME=sec.)	=	2.20 in
h_o=	TAB("AISC/W";h_o;NAME=sec.)	=	9.30 in

(r_{ts} is the Effective radius of gyration for the L.T.B. and h_o is distance between C.L. of flanges)

AISC Specification Eqn. (F2-1), yielding Moment in major axis (M_{px}):

M_{px}= $Z_x*F_y*1/12$ = 162 kip*ft

AISC Specification Eqn. (F6-1), yielding Moment in minor axis (M_{py}):

M_{py}= MIN ($Z_y*F_y*1/12;1.6/12*S_y*F_y$) = 58 kip*ft

Required Flexural Strength (including second-order amplification)

Use the approximate method of second-order analysis procedure from AISC Specification Appendix 8.

Because the member is not subject to sidesway, only P-δ amplifiers need to be added.

$$B_1 = \frac{C_m}{1 - \alpha * P_r / P_{e1}} \geq 1 \quad \text{(Spec. Eq. A-8-3)}$$

The x-x axis flexural magnifier is,

C_{mx}= 1.00

$$P_{e1} = \frac{\pi^2 * E * I_x}{(kL_{in} * 12)^2} = 1734 \text{ kips}$$

α= 1.00

$$B_{1x} = \frac{C_{mx}}{1 - \alpha * P_u / P_{e1}} = 1.02$$

M_{ux}= B_{1x}* M_{ux1} = 91.8 kip*ft

The Y-Y axis flexural magnifier is,

P_{e2}= $\dfrac{\pi^2 * E * I_y}{(kL_{out} * 12)^2}$ = 371.2 kips

C_{my}= 1.00

B_{1y}= $\dfrac{C_{my}}{1 - \alpha * P_u / P_{e2}}$ = 1.09

M_{uy}= B_{1y}* M_{uy1} = 13.1 kip*ft

Element Classification

(1) Web:

h/t_w, λ_w= TAB("AISC/W";h/t_w;NAME=sec.) = 27.10

According to AISC Specification Table B4.1 Case 9, the limiting width-to-thickness ratio for the web is:

Web_Class= IF($\lambda_w \leq 3.76 * \sqrt{(E/F_y)}$);"Compact"; "Non-Compact") = Compact

(2) Comp. flange:

$b_f/2t_f$, λ_f= TAB("AISC/W";$b_f/2t_f$;NAME=sec.) = 9.15

According to AISC Specification Table B4.1 Case 1, the limiting width-to-thickness ratios for the compression flange are:

λ_{pf}= $0.38*\sqrt{(E/F_y)}$ = 9

λ_{rf}= $1.00*\sqrt{(E/F_y)}$ = 24

Fl_Class= IF($\lambda_f \leq \lambda_{pf}$;"Compact";IF($\lambda_f > \lambda_{rf}$;"Slender";"Non-Compact")) = Non-Compact

The available strength provided by AISC Specification Sections F3.1, F3.2, F6.1 and F6.2, the nominal flexural moments in strong/weak axes are calculated as follows, satisfying the condition of compression Flange Local Buckling:

M_{nx1a}= M_{px}-0.7*F_y*S_x*1/12 = 60 kip*ft

M_{nx1}= IF(Fl_Class="Compact"; M_{px}; (M_{px}-M_{nx1a}*$\left(\dfrac{\lambda_f - \lambda_{pf}}{\lambda_{rf} - \lambda_{pf}}\right)$))) = 161 kip*ft

M_{ny1a}= M_{py}-0.7*F_y*S_y*1/12 = 31 kip*ft

M_{ny1}= IF(Fl_Class="Compact"; M_{py}; (M_{py}-M_{ny1a}*$\left(\dfrac{\lambda_f - \lambda_{pf}}{\lambda_{rf} - \lambda_{pf}}\right)$))) = 58 kip*ft

Slenderness Check (According to section E2)

For members designed on the basis of compression, the slenderness ratio KL/r preferably should not exceed 200.

λ_x= $\dfrac{kL_{in}}{r_x}$*12 = 40.1

λ_y= $\frac{kL_{out}}{r_y}$*12 = 86.6

Then, the governed slenderness (λ_{max}):

λ_{max}= MAX(λ_x;λ_y) = 86.6

Critical Stresses

The available critical stresses may be interpolated from AISC Manual Table 4-22 or calculated directly as follows:

-Calculate the elastic critical buckling stress, F_e:

F_e= $\frac{\pi^2 * E}{\lambda_{max}^2}$ = 38.2 ksi

-Calculate the flexural buckling stress, F_{cr} (Eqns. E3-2 and E3-3):

λ_1= 4.71*√(E/F_y) = 113

F_{cr}= IF(λ_{max}≤λ_1; $(0.658)^{\left(\frac{F_y}{F_e}\right)} * F_y$;0.877*$F_e$) = 28.9 ksi

Design Compressive Strength (Eqn. E3-1)

P_n= F_{cr}*A = 281 kips

Φ_c= 0.90

$\Phi_c P_n$= Φ_c*P_n = 253 kips

Lateral Torsional Buckling (LTB)

The limiting lengths Lp and Lr are determined according to the AISC Spec. Eqns. F2-5 and F2-6, as follows:

L_p= 1.76*r_y*√(E/F_y)/12 = 6.85 ft

L_{r1}= $\sqrt{\frac{J * 1.0}{S_x * h_o}}$ = 0.04

L_{r2}= $\sqrt{1 + \sqrt{6.76 * \left(\frac{0.7 * F_y * S_x * h_o}{E * J * 1.0}\right)^2}}$ = 1.66

L_r= 1.95/12*r_{ts}*$\frac{E}{0.7 * F_y}$*L_{r1}*L_{r2} = 19.67 ft

Case= IF(L_b>L_r;"ELTB";IF(L_b≤L_p; "No LTB";"InLTB")) = InLTB

("ELTB" refers to elastic LTB and "InLTB" refers to inelastic LTB)

According to the AISC Spec. Eqn. F2-2:

M_{1a}= M_{px}-0.7*1/12*F_y*S_x = 59.9 kip*ft

M_1= MIN(M_{px}; C_b*(M_{px}- M_{1a}*(L_b-L_p)/(L_r-L_p))) = 147 kip*ft

According to the AISC Spec. Eqn. F2-4:

F_{cr}= $C_b*\pi^2*E/((L_b+0.01)*12/r_{ts})^2$ = 55.87 ksi

$F_{cr,mod}$= $\sqrt{(1+0.078*J*1.0/(S_x*h_o)*(L_b*12/r_{ts})^2)}$ = 1.35

According to the AISC Spec. Eqns. F2-3:

M_2= $MIN(M_{px};F_{cr}*S_x/12*F_{cr,mod})$ = 162 kip*ft

According to the AISC Spec. Eqn. F2-2:

M_{nx2}= IF (Case="No LTB";M_{px};IF(Case="InLTB";M_1;M_2)) = 147.0 kip*ft

Design Flexure Moment in Major/Minor Axes

Φ_b= 0.90

M_{nx}= $MIN(M_{px};M_{nx1};M_{nx2})$ = 147.0 kip*ft

M_{ny}= $MIN(M_{py};M_{ny1})$ = 58.0 kip*ft

Calculate The Available Flexural and Axial Strengths

P_c= $\Phi_c * P_n$ = 252.90 kips

M_{cx}= $\Phi_b * M_{nx}$ = 132.3 kip*ft

M_{cy}= $\Phi_b * M_{ny}$ = 52.20 kip*ft

Check The Combined Stress Ratio (AISC Specification Section H1-1a and H1-1b)

Axial ratio, p= $\frac{P_u}{P_c}$ = 0.12

Moments ratio, m= $\frac{M_{ux}}{M_{cx}} + \frac{M_{uy}}{M_{cy}}$ = 0.94

Stress_ratio= IF(p≥0.2; (p+8/9*m);(p/2+m)) = 1.00 in

Safety= IF(Stress_ratio≤1;"Safe";"Unsafe") = Safe

Design Summary

P_c= $\Phi_c * P_n$ = 252.9 kips

M_{cx}= $\Phi_b * M_{nx}$ = 132.3 kip*ft

M_{cy}= $\Phi_b * M_{ny}$ = 52.2 kip*ft

Stress_ratio= IF(p≥0.2; (p+8/9*m);(p/2+m)) = 1.00

Safety= IF(Stress_ratio≤1;"Safe";"Unsafe") = Safe